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ABSTRACT

The facets of teachers' views of mathematics include their personal views, how they view mathematics for their students, how they view the mathematics curriculum, and how they view mathematics teaching. This paper reports a pilot study to examine how elementary school teachers view mathematics both personally and for their students. A coding scheme was utilized to identify 22 elementary school teachers' views of knowledge about mathematics and purpose of schooling. When put into a matrix, the intersection of the views of knowledge and the purpose of schooling created four categories that defined a teacher type: Conveyor, Organizer, Allower, and Facilitator. Of the 22 teachers, 5 representing each of the categories and a diversity of grade levels were chosen to be interviewed. Profiles developed from the interviews are presented for each of the five teachers. Findings indicated that 13 out of the 22 teachers were classified as conveyors. Consistency was found for four out of the five teachers in how the teachers viewed mathematics, what they stated students should know, and their placement within the coding scheme. Although elementary school teachers' views of knowledge of mathematics are reflected in what they believe is important for their students to learn, the relationship between those views and what is taught is not always clear. (Contains 26 references.) (MDH)

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# Elementary School Teachers' Views of Knowledge Pertaining to Mathematics

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## Elementary School Teachers' Views of Knowledge Pertaining to Mathematics

What is mathematics and how is it known? The answer to this question varies widely and is not always consistent. According to Davis and Hersh (1981), a naive definition, yet adequate for an initial understanding, is "mathematics is the science of quantity and space" and the related symbols (p. 6). Another definition is mathematics is the "science of patterns" and no longer just the study of space and number (Steen, 1988). In addressing this question, Romberg (1992) addresses mathematics first philosophically by distinguishing between absolutist and social constructivist stances, as well as what is meant by knowledge, the aspects of culture, and the actual "doing" of mathematics. For Romberg, a major difference in mathematical knowledge is "knowing that" meaning the body or record of the discipline as compared to the "doing" or "knowing how" which is the active constructing of knowledge. Many educational researchers have come to the conclusion that mathematics as taught is typically "knowing that" or skills and concepts to be mastered and verified as correct by the teacher (e.g., Lampert, 1990; Romberg, 1992; Stodolsky, 1985; etc.)

In considering what math is and how it is known, several different aspects of understanding arise. Teachers' views of mathematics are multi-layered and include how they personally conceive of mathematics, how teachers view mathematics for their students, how they view the mathematics curriculum, and how teachers view what and how to teach. These areas are not necessarily exclusive, and at times, considerable overlap exists. Based on a pilot study addressing the multi-layered aspects

and interconnections of teachers' views of knowledge in mathematics, this paper explores how five elementary school teachers view mathematics both personally and for their students.

## Research Strategies

A variety of orientations exist to interpret how teachers approach teaching, especially from the various social and behavior sciences. A small, but growing literature attempts to explain views of knowledge or epistemological stances, especially pertaining to a particular subject area such as math or various sciences (e.g., Carey, Evans, Honda, Jay, & Unger, 1989; Greeno, 1988; Schoenfeld, 1989). The research strategies for this study include: 1) identifying teachers with diverse views of knowledge about mathematics using a coding scheme and written responses to two prompts; and 2) interviewing five teachers selected for their diversity of placement within the scheme.

The coding scheme combines the concepts of teachers' views of knowledge and classroom teaching within the field of math. The scheme developed comprises two dimensions each having two categories within. The first dimension is the orientation of the teacher viewing knowledge as primarily content to be taught or process to be learned. The second dimension is the orientation of the purpose of schooling being school knowledge or child development. The basis for breaking the categories in this fashion is supported by literature in development, teaching, and curriculum.

The first dimension, knowledge as content or process, is situated in work done by Perry, (1970, 1981) and Belenky, Clinchy, Goldberger, & Tarule (1986). Perry (1970) developed a nine position continuum ranging

from Authority (often the professor) and Absolutes (capitals in original) to the acceptance of a relative world, where commitment and balance are developed, through the alternation of reflection and action. Just prior to reaching Position 5 is a shift from "what" or the content of coming to know, to the "way" or generalized process by which something is known (Perry, 1981, p. 88).

Building on Perry's (1970) scheme and from extensive interviews with women, Belenky et al. (1986) describes epistemological perspectives from which women know and view the world. Within their continuum of five major categories, a significant shift occurs between the second category (received knowledge) and third category (subjective knowledge) when a woman changes from being a receiver of knowledge to a person who perceives knowledge to be personal and subjectively known. This shift again reflects the difference between knowledge as content and knowledge as a process.

When considering elementary school math, content oriented teachers emphasize activities and/or exercises without connecting what is learned or the interrelatedness of various forms of math. The emphasis is "what" is known, often as separate entities, such as basic math facts, procedures for long division, what unit to use for measurement, etc. Those teachers who stress the process(es) of building broad applications and structures, or extensive interacting toward the purpose of creating meaning and understanding are categorized as process oriented. Knowledge as a process includes more of the "how" and ways of coming to know, often emphasizing ideas such as frameworks, negotiation of meanings, and the interrelatedness of various aspects of math.

The second dimension, the purpose of schooling, divides into the two categories of school knowledge and child development. Metz (1978) found

this distinction in the teachers she studied. Those who were "incorporative" believed in teaching subjects while those teachers who stressed "developmental" education were interested in teaching children. Jackson (1986) clarifies the differences between school knowledge and child development by "mimetic" (presented, passed on, reproducible, and measurable knowledge) and "transformative" (accomplishing a qualitative change of some kind in the person being taught).

The difference between school knowledge and child development lies in where the initial emphasis is placed by the teacher. Within the classroom, those teachers who start with some form of the curriculum, be it the textbook, district curricular guides, or state frameworks, would be classified within the school knowledge dimension. In contrast are those teachers who start by considering the children's needs and interests, how the children learn, their unique backgrounds, etc., who would be classified within the child development dimension.

When placed into a matrix, the intersections of the views of knowledge and the purpose of schooling create four categories which define a type of teacher (Table 1). Each category is more fully developed below.

Table 1

| Purpose of Schooling | Knowledge as Content | Process |
|---|---|---|
| School Knowledge | Conveyor | Organizer |
| Child Development | Allower | Facilitator |

Within the first category is the teacher who is a Conveyor. This person defines math knowledge as procedural and accumulative, to be taught by the teacher as expert, and the students as receptacles and memorizers. The Conveyor is represented in the literature as one who tells, followed by the students doing math practice (e.g., Stodolsky, 1985, 1988), an "executive" (Fenstermacher & Soltis, 1986) or a proficient information provider (Prakash & Waks, 1985).

The Organizer, while oriented toward school knowledge, perceives math differently than the Conveyor in that this teacher strives to teach students some kind of framework, scheme, or theory that assists students in connecting their math knowledge so that the students are able to apply the core concepts successfully. Cognitive models such as "information processing" approaches (Joyce & Weil, 1986) or concept development (Taba, 1966) are indicative of the Organizer.

Along the dimension of child development, the Allower relies on activities to teach the students. The teacher provides the activities, and from participating in the activities, individual growth and development are nurtured. Teachers in this category may be perceived as "therapists" (Fenstermacher & Soltis, 1986) or as assisting with self-actualization (Prakash & Waks, 1985).

The Facilitator, while emphasizing the child, strives toward constructed and shared meaning acquired through joint inquiry and negotiation. The teacher is a facilitator of conversations and interactions, and the students are responsible for constructing new ideas, connections, meaning, and arriving at consensus (see Duckworth, 1987; Lampert, 1990).

To begin delineating more precisely what each teacher category included, generic questions were formulated then answered according to each teacher classification. The first two questions were what is knowledge and how is knowledge acquired/gained.

To further develop and make these categories operational, elements from Dillon's (1988) classification scheme of the classroom were combined with the two dimensions of the purpose of schooling and knowledge as content or process. One element is subject matter. Since the subject matter was limited to math, no further questions were developed concerning that specific topic. However, in considering the first question (what is knowledge), the presumption is that knowledge pertains to math and therefore is actually part of the category of subject matter. Another of Dillon's categories is activity (of both teacher and student). This is taken in a broad sense and incorporated into the question of how is knowledge acquired/gained. Other categories from Dillon are teacher, student, aims or purpose (toward what end?), and results (how do you know?). By using Dillon's classification system, generic questions were used to develop a matrix which served as the basis for the categorization of the teachers (Appendix 1 and Appendix 2).

Two prompts, to be responded to in writing, were designed to provide the data for the categorization of the teachers (Appendix 3). The first prompt asked for a description of a very good math lesson taught by the teacher and requested what the teacher and students did be included. The purpose of this prompt was to aid in the differentiation between knowledge as content or process. The second prompt asked the teacher to identify what he/she hoped the students learned by the end of the year in math. This response was especially directed toward distinguishing the purpose of schooling.

Both prompts were designed to differentiate among the six classification questions leading to the categorization of Conveyor, Organizer, Allower, and Facilitator.

## Teachers' Views of Mathematics and Teaching

In order to identify elementary school teachers with diverse views of knowledge of math, a total of twenty-two teachers, grades kindergarten through six, representing eight school districts, were recruited. Each volunteer responded to two prompts by writing their answers. The teachers were categorized according to their written responses and the coding scheme developed. The totals for the classification categories are shown in Table 2.

Table 2

| | |
|---|---|
| Conveyor (13) | Organizer (5) |
| Allower (1) | Facilitator (3) |

A total of 5 males and 17 females participated. All 5 males were classified as Conveyors. Their years of teaching experience ranged from 1.5 to 19 with a mean of 7.8 years. For the 17 females, 8 were Conveyors, 5 were Organizers, 1 was an Allower, and 3 were Facilitators. Their years of teaching experience ranged from 2 to 19 with a mean of 7.6 years.

Grade level distributions, gender, and classification can be found in Table 3.

Table 3

**Females**

| Grade | K | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| N | 1 | | 2 | 5 | 3 | 2 | |
| Classification | (O) | | (FC) | (4C1A) | (2O1F) | (2C) | |
| Mixed Grades | | 1-2 | | 2-3 | | 3-4-5 | 5-6 |
| N | | 1 | | 1 | | 1 | 1 |
| Classification | | (C) | | (F) | | (O) | (O) |

**Males**

| Grade | K | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| N | | | | 1 | 1 | | 2 |
| Classification | | | | (C) | (C) | | (2C) |
| Mixed Grades | | K-1-2 | | | | | |
| N | | 1 | | | | | |
| Classification | | (C) | | | | | |

C = Conveyor O = Organizer A = Allower F = Facilitator

The written prompts and the coding served to identify teachers with diverse views. From the coding data, five teachers were selected to be interviewed about the ways they think about math, their ways of teaching math, and their understandings of their students' ways of learning math. The five teachers selected were a female kindergarten teacher (Organizer with many aspects of a Conveyor), a female combination grades two-three teacher (Facilitator), a female third grade teacher (Allower), a female

combination grades three-four-five teacher (Organizer), and a male sixth grade teacher (Conveyor). This selection provided a teacher from each category and a diversity of grade levels were represented.

The names of all the teachers have been changed to preserve their anonymity. The speech, however, has not been changed and is reported here as representative as possible.

Gene

Gene, categorized as a Conveyor, is a sixth grade, second year teacher, and is just short of a college math minor. For him, math is computation and did not become applicable in the real world until calculus:

> Math is? I, uh, (pause) to me it is, I know I could come up with some kind of great answer, you know, but, it's, it's just computation, you know, pretty much. Now when I got, when I got into calculus, that was the first time that, that math, I could, I could relate math to the real world, because then we could, we could take uh, you know, suspension bridges and, and, things like melting ice cubes and things like that, you know, that I could, I could see a purpose and use for it.

Gene goes on to say that he does try to show the students how math is applicable to the world but, "it's real hard for me to relate it to real life for the kids and in my mind it's, math is problems that, you know, you just have to, you know, to compute the answer on." He does acknowledge that math can be "real" for the students:

> I tease them, whenever we have to figure out uh, if we're having a pizza party and we've got four pizzas and we've got thirty-two kids, and you know, how're we gonna figure this out.

However, personal application at this level is peripheral to Gene and not the focus nor the purpose of math.

He distinguishes between his own understanding of what math is, and what is important for students to know. Math is:

> ...special because...it's an objective thing and you can always count on two plus two being four and there's a comfort in that. I found that I started going through the higher math and it was hard for me, and it was a challenge, but once you've got it, it was, there was a real sense of accomplishment, you know. ...But, I guess for, what I expect of them and that type of thing and in the sixth grade, you know, see I, I'm not sure how to answer. ...So, it's, it's mostly need to learn the basic skills and things like that to do well in junior high school and high school."

Gene also stated that math is "one subject that's fairly cut and dry...they have their paper, they have their problems and they need to do it."

For Gene personally, to know math is to be able to compute, apply "higher" math to real life situations, and to master the material. For his students, math is learning the basic skills to do well in future grade levels. Gene wants his students to succeed, to master the material, and, as it was for him as a young student, not be concerned about math having a purpose in their own lives.

Judy

Judy, categorized as an Organizer with many Conveyor aspects, is a fifth year kindergarten teacher. Judy places math within the setting of her classroom when she states:

> ...I think math is the concept of numbers that deal with the kindergarten, you know, life. What affects them. I don't think math to them is anything beyond that. ...I think that um, at least math for me in this setting is what is relevant to kindergarten and that's basically the things they have to deal with. Counting, understanding uh, numbers, and being able to write numbers.

For Judy, to know mathematics is "to be able to function, to have those things that you need to know." For her personally, "being able to add and subtract, I need to know that. Kindergarteners don't. Being able to multiply, um, whatever I need, that's what it means to me." For herself, Judy states:

> I know I use it everyday. And if I sat down and count up all the different things I'd probably be surprised. But, um, math, I, I think for me is a, my checkbook and my calendar and ah, timeframes at school.

For Judy, there is a difference between knowing mathematics and thinking mathematically. Knowing math is to be able to function by knowing the things a person needs to know, while someone who thinks mathematically "is somebody who's very analytical. And somebody who, um, has to um, really, ah, know all the cause and effect approaches to something and is very um, cut and dry about a lot of information. So they, they just seem very analytical."

She also does not view math as a separate discipline area within her kindergarten class. She sees math as integrated within the curriculum throughout the day such as:

> ...we do the calendar, we count the boys, count the girls, put up the number every day. We might be counting something else during the day. We might be talking about how many of this or that, we might be talking about time, in ten minutes we're going to do something or, you know, so there's a lot, in kindergarten it's, it's all integrated.

In fact, "math" is not a common nomenclature for her students. Judy states "calling it math to kindergarten is kinda basic, sometimes I say you're going to do math now and they look at me like we're going to do what?"

Judy views math as functional and integrated into each day. To know math is to be able to use it in a person's daily life.

Jackie

Jackie, categorized as an Allower, has been teaching third grade for the last four of her eight years of teaching. Mathematics to Jackie is "a lot of logic and reasoning, ...figuring things out, being able to, to take in a lot of different, um, information and just basically figuring out the best answer the

way, you know, the right answer, you know, being able to process information." However, in the following sentence, Jackie begins to give a rationale between what math is and what is taught:

> That's not always the way it, you teach it, you know. We, well, you know, just because I, you know, there is so much curriculum that you have to get through, but you don't always have the time to say, to, to be able to present that to the kids that this is, you know, taking this and taking what you know and, and processing it. So, um, you know, there's a lot, there's just a lot to get through. The kids have, you know, basic facts left from now on, which are important and I wish the parents would do more with that so I could take less time on, on that kind of stuff, you know, and, and do the real, the real stuff.

The "real stuff" to Jackie is "taking in real life situations" and "making math, ah, a more real part of their lives." In summarizing what math is, Jackie states that it is "being able to...apply numbers to the solving of problems...in everyday situations." To know math is to "actually use those numbers in a concrete way...being able to go outside of the school arena...figure things out on their own."

The "real stuff" for Jackie is using numbers to solve problems in everyday situations. However, she feels a discrepancy between her view of math and what she teaches. When asked what is important for the students to learn, she replied:

> ... making sure I get through the curriculum, there's such a, an emphasis on testing. I wouldn't have said that you know, five or six years ago and I wouldn't have said that at all, just you know, getting them to realize that they're going to use math in the real world and they're you know, and I still would like to be able to do that, but there is such an emphasis on testing for teachers....

Jackie returns to the issue of what is relevant for a third grader to know as compared to what is tested:

> ...there are things honestly, that we test in third grade and that they demand on most of those standardized tests that these kids know or, or something and, when does a third grader really going to need to know about volume? And those two pages they give you in the math

> book, or area, really, is this a necessary concept for a third grader at this point? You know, I, I would like to be able to, and I don't know why I don't have the guts to do, go through the math book and figure what is necessary, what is not necessary.

Jackie is cognizant of the discrepancies of what math is to her, what she believes a third grader should know, and the curriculum as established by the text and standardized testing.

Sherri

Sherri, a Facilitator, teaches a combination grades two-three class and is in her fifth year of teaching. For Sherri, math is "many things. It teaches us discipline, it teaches us, um, procedures, um, logic, and how to think." In expanding what she means, Sherri states that discipline means that "you have to learn your steps, your formulas" because "if you don't go through those steady paces and learn those skills, you can't build. And it does take discipline to do it." Procedures are different because you can skip a step, and logic "comes into a part of discipline and stuff, how to make the mind look at things logically." How to think involves having "to think about the steps you're doing. And you have to concentrate on that. And that's part of it. Um, there's something that you can do without thinking, but math is not one of them."

What is important for the students to learn is:

> Recognizing numbers, for one. Understanding the concept or the um, the symbols of what some, you know, what the numbers represent. ...I think that's important to know your numbers um, and concept, understanding of the symbols real well. ...I would like them to um, be able to problem solve. To think for themselves, you know. ...being able to, to figure things out. I think that's more important than anything right now.

Sherri's approach to teaching and understanding mathematics is by focusing on the children and having the students learn from each other.

> So it's more individualized than whole class. Um, so I teach, I'm not the center, the children are the center, I'm the guide. ...I don't need to teach the whole class, because some of them learned it, um, some of them are learning from each other....

Sherri's role as a teacher is to "guide. Be a helper." A teacher should "also be able to ask the right questions to get them to find the answer, not just give them the answer, um, so that they can solve it for themselves." The hardest thing about teaching math is "(p)utting it in terms they understand. Just because you know how to do it, and you understand how to do it, doesn't mean you can tell them how to do it." For Sherri, knowledge in math is not in the telling; it is in the construction of meaning for each student.

Mary Beth

Mary Beth, an Organizer, responded to what is math in a global sense. She teaches a combination grades three-four-five class, and is in her second year of teaching. She views math as problem solving, thinking and reasoning, and that it is logical. In expanding on what she meant by problem solving she stated:

> Um, problem solving to me is, is you're presented with a problem, whether it be a physical or a thinking or something, something that you need to work through to come out with a solution, and you know, if, to me problem solving is, can be tied into all subject matters and should be, it's just that ability to, you know, we talk about critical thinking, yeah, but it's that ability to take the information you already know and apply it to a new situation and to pull resources when necessary to come to a conclusion to um, to work out a situation.

For Mary Beth, mathematics is problem solving, to know mathematics is to be able to problem solve and to think mathematically is when someone is "logical...but not always," and "trying to fall into some step by step process for, for, um, problem solving that is, is a mathematical way of going at it."

What is important for students to learn in Mary Beth's class are problem solving techniques of "how to work through figuring something out," basic facts and operations, vocabulary, and checking for sensible answers. She goes on to say:

> I do think it's important that they see a, an application in their life for any of the rest of that to be, you know, I think that if they're inspired that God, this is going to help me in my future, then they're going to be good problem solvers, and if they're good problem solvers, they're going to be able to acquire basic facts, and, and work through them and understand what they mean for them. So I think it is all interrelated.

Mary Beth views real life application as an essential part of math. She stresses "why they need to know these things" through activities and experiences such as cooking and converting measurements, checking accounts and budgeting, and interviewing parents about how they use math in their everyday lives and in their careers. She strives to make math meaningful and useful to her students, usually through setting up some sort of problem for her students to solve.

## Findings

This pilot study found consistency for four of the five teachers in how they view mathematics, what they state students should know, and their placement within the coding scheme. However, each teacher is unique and emphasizes different aspects of mathematics.

Gene, in his efforts to complete the text and help his students master the material, strives toward his conception of mathematics as computation. Gene focuses on abstractness and not on any of the other three features of mathematics curriculum of inventing, proving, or applying (Romberg, 1992). In fact, application to "real life for the kids" is very difficult for Gene. As a

Conveyor, Gene indeed strives to impart procedural mathematical knowledge. Algorithms and basic facts are practiced daily so that students might progress at grade level expectations.

Math is well integrated into the curriculum for Judy, just as it is embedded in her own life. She looks for the tools necessary to function at a given point in life, be it as a kindergartener or as an adult. Judy, as an Organizer with many Conveyor aspects, stresses number value and concepts with her kindergarten students through multiple integrated experiences. Along with the emphasis on number concepts Judy also repeatedly practices counting, number writing, identifying coins and other basic nominal tasks.

For Sherri, math teaches discipline, procedures, logic, and how to think. She places the emphasis on process, especially of understanding and being able "to figure things out." Sherri's emphasis on student-centered learning and developing the students' abilities to problem solve are indicative of her placement as a Facilitator. She is commited to asking "the right questions" and guiding students in their own inquiry.

For Mary Beth, math is meaningful and useful through problem solving personally and for her students. Mary Beth strives to create structures and the means for solving problems through multiple experiences. Actual and meaningful mathematical encounters challenge her students to apply what they are coming to know.

The one teacher who is not consistent in how she views mathematics and what she currently states as what her students should know is Jackie. Jackie currently views teaching the text as a means of meeting the criteria necessary for higher test scores, but questions the relevance for her students. For Jackie, math is being able to use numbers to solve problems

in real life. "Getting through the curriculum" as viewed by Jackie does not allow enough time to do the "real stuff." For Jackie, the initial placement as an Allower is an enigma. From her interivew, she clearly espouses the characteristics of a Conveyor in her efforts to "get through the curriculum." However, she strongly questions the relevance of what is taught in relation to what is important and necessary for a third grader to know. She is caught between the emphasis on school knowledge and child development.

## Conclusion

Elementary school teachers' views of knowledge of mathematics are reflected in what they believe as important for their students to learn and what is taught. However, in this pilot study, this relationship is not always consistent.

Numerous studies (e.g., Lampert, 1990; Romberg, 1992; Stodolsky, 1988; etc.) have lamented the current state of affairs in the teaching of mathematics as transmissionist and uninspiring. To answer this lament, more social constructivist approaches for mathematics education are being advocated (e.g., National Council of Teachers of Mathematics, 1989; Romberg, 1992; Schoenfeld, in press; etc.). However, the critical aspects of social and cultural pressures have been neglected in much of the literature.

Lampert (1990) has directly challenged the usual social interaction roles and patterns of discourse within the elementary classroom. Her purpose in doing so was to examine whether it is possible to bring knowing mathematics in school more in line with the knowing mathematics within the discipline. She is left not knowing what knowledge her students actually have.

The results of approaches to teaching mathematics such as those practiced by Lampert (1990) are difficult to know in the traditional sense. As long as teachers feel pressured by social factors such as standardized test scores, a basic conflict arises. If schools and teachers are judged by the results of standardized test scores, yet approaches to teaching mathematics become more in line with what it means to truly know mathematics where the results cannot be guaranteed nor measured by standardized testing, the dilemma as experienced and so poignantly expressed by Jackie in this study will continue to be faced by classroom teachers.

Although no claims are made for generalizability from this study, it is striking that 13 of 22 teachers were classified as Conveyors and that the average number of years teaching for this category is 12.1 with a range from 1.5 years to 19 years. The argument can be made that social pressures, such as test scores, support this mode of knowing and teaching mathematics in our schools, since the Conveyor is the most in line with what is tested including factual recall, algorithmic applications, and short procedural tasks.

Many questions remain. One question to further pursue is how broad issues of social and cultural factors affect classroom teachers' views of knowledge. Other questions to address in future research include exploring and analyzing how stable teachers' views are in other subject areas, over time (including different times of the school year and having different groups of children each year) and as they change grade levels, curricular frameworks, administrators, and schools. The realm of what is taught as perceived by the teachers would add another dimension to this research, both from self-reports, and from classroom observations.

Interest in teachers' thinking and beliefs has rapidly grown during the last decade. More recently, the connection between mathematics and

teachers' thinking, especially at the elementary level, has been explored in diverse ways such as understanding how teachers understand children's thinking about mathematics (e.g., Carpenter, Fennema, Peterson, & Carey, 1988), teachers' thinking about problem solving (Thompson, 1988), and knowing mathematics and multiplication (Lampert, 1986). There still remains an "absence of substantial evidence about the relationship between teachers' conceptions of a field and their representations of it to students" (Schrag, 1992, p. 290). This research serves as another piece in understanding connections that teachers make between what they perceive math to be and what students are to learn.

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## Appendix 1

## Matrix of Teacher Categories

For the teacher as... (dimension of school knowledge)

| | Conveyor | Organizer |
|---|---|---|
| What is knowledge? | Accumulation of facts and algorithms<br>Procedural | Acquisition of schemata, frameworks and/or structures<br>Theory driven |
| How is knowledge acquired/gained? | Conveyed by expert, authority<br>Observed<br>Drill and practice of basic facts and algorithms | Expert presentations of underlying structures<br>Multiple experiences leading to application of structures |
| Toward what end? (purpose) | Master facts, algorithms<br>Progress at grade level | Master core concepts<br>Create cognitive map |
| How do you know? (results) | Right answers to problems, confirmed by authority | Students able to apply theory, and transfer to new situation |
| Teacher as... | Conveyor, teller, transmission specialist;<br>Authority and expert | Organizer and planner of experiences to foster meta-cognitive growth |
| Student as... | Receptacle<br>Memorizer<br>Participant in exercises | Method seeker<br>Understander and applier of broad concepts |

For the teacher as... (dimension of child development)

| | Allower | Facilitator |
|---|---|---|
| What is knowledge? | Experienced intuition | Constructed and shared meaning |
| How is knowledge acquired/gained? | Participation in activities, often self-selected and individual | Inquiry<br>Interactive negotiation |
| Toward what end? (purpose) | Individual growth and development | Construction of new ideas<br>Interrelatedness of knowledge |
| How do you know? (results) | Student engagement in activities | Negotiated consensus |
| Teacher as... | Allower and provider of activities | Facilitator of conversations |
| Student as... | Experiencer of activities | Autonomous thinker socially able to express and substantiate own views and rationales |

## Appendix 2

## Expanded Notation on Coding Scheme

### CONVEYOR

**School Knowledge** is the purpose of schooling
Knowledge as **Content**

**What is knowledge?** Accumulation of facts and algorithms
Procedural

EX. I want them to know all of the basic facts.
Students will be able to multiply 2 x 3 digit numbers

**How is knowledge acquired/gained?** Conveyed by expert, authority
Observed
Drill and practice of basic facts and algorithms

EX. Using the book, the students did 10 subtraction problems.
I showed them how to do long division by working out a problem on the overhead.

**Toward what end? (purpose)** Master facts, algorithms
Progress at grade level

EX. By the end of fourth grade, all the students should know their multiplication facts.
I want to cover all of the stuff in the book this year.

**How do you know? (results)** Right answers to problems, confirmed by authority

EX. We checked all of our answers together and they put the score at the top of their papers.
They have to get 80% on the end of the chapter test to pass.

## ORGANIZER

**School Knowledge** is the purpose of schooling
Knowledge as the **Process**

**What is knowledge?** Acquisition of schemata, frameworks and/or structures
Theory driven

EX. By the end of the year, I want them to understand concepts of numbers and not just how to add and subtract.

**How is knowledge acquired/gained?** Expert presentations of underlying structures
Multiple experiences leading to applications of structures

EX. They first created a chart of all the multiples of 9 up through 25. Then they looked for patterns that the numbers created. Then each group picked 2 other numbers to do the same thing and compare their findings.

To demonstrate 2 digit multiplication, I showed them how to decompose the numbers first, e.g., 23 is actually 20 + 3. Then I showed them how each part is actually multiplied and added, and compared the process to the "regular" way of doing it. By working several problems this way, they began to see how multiplication is more than just doing the steps.

**Toward what end? (purpose)** Master broad concepts
Create cognitive map

EX. By the end of the year, I hope that they have an understanding of fractions as kinds of ratios, as well as being able to reduce and find the lowest common denominator.

Besides the basic facts, I want them to understand how addition and subtraction are related, and multiplication and division, and addition and multiplication, and subtraction and division.

**How do you know? (results)** Students able to apply theory, and transfer to new situation

EX. They really surprised me when they figured out the problem, especially since it involved dividing by two digits. They're only third graders!

## ALLOWER

**Child Development** is the purpose of schooling
Knowledge as the **content**

**What is knowledge?** Experienced intuition

EX. When he was playing with the blocks, you could just see the wheels turning.

**How is knowledge acquired/gained?** Participation in activities, often self-selected and individual

EX. During free time, they pick what center they want. I put different math manipulatives out each day.
Everyone's always doing something, but they might not be doing the same things.

**Toward what end? (purpose)** Individual growth and development

EX. By the end of the year, I would like everyone to feel comfortable with math Everyone progresses at their own rate.

**How do you know? (results)** Student engagement in activities

EX. I really want everyone to be involved.

## FACILITATOR

**Child development** as the purpose of schooling
Knowledge as the **process**

**What is knowledge?** Constructed and shared meaning

EX. Through the development of a classroom community, I want them to come to understand that they're the ones who figure this stuff out. They need to talk about it and decide how to solve it and if it's right.

**How is knowledge acquired/gained?** Inquiry
Interactive negotiation

EX. When a student comes to me to ask how to do a problem, I ask her what she wants to know. Then I ask her what she already knows. Finally, I try to get her to ask the kinds of questions that will help unravel the problem so she can pursue it.

The students learn from each other by talking about their work. They take a situation and work on how to set it up, if it's going to work, and all the attempts trying to figure it out.

**Toward what end? (purpose)** Construction of meaning
Interrelatedness of knowledge

EX. By the time they were done, they used intuitive, concrete, and conceptual ways of solving the problem.

I didn't tell them how to do it. But after working together and talking about it, they knew how to add unlike fractions.

**How do you know? (results)** Negotiated consensus

EX. I'm not the ultimate authority here. They discuss it and decide if it's right.

## Appendix 3

## Writing Prompts

Name _____

Years of teaching experience _____

Current grade level taught _____

Previous grade levels taught _____

Education beyond teaching credential _____

_____

On each of the following two pages you will be asked to respond to a statement.

Take about five minutes or so for each page. I will let you know when five minutes and then when ten minutes have passed. You are welcome to write longer if you would like.

Please write as clearly as possible.

Please wait until asked to begin.

Thank you!

Describe one math lesson you have taught that you thought was very good.
Be sure to include what you did and what the students did.

For math---By the end of the school year, I hope my students have learned...